We begin with a "trivial" condition on massive gravitons, and ask now how this condition affects future efforts in early universe GW investigations. Forefront of research questions investigated with massive gravity, and emergent space-time

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Abstract

The methodology is simple. Use a construction for a minimal time-step, then from there get emergent space-time conditions for a bridge from a nonsingular start to the universe, to potential Quantum gravity conditions.

Our Methodology is to construct using a "trivial" solution to massive gravitons, and a nonsingular start for expansion of the universe. Our methodology has many unintended consequences, not the least is a relationship between a small timestep, t, the minimum scale factor and even the tension or property values of the initial space-time wall, and that is a consequence of a "trivial" solution taking into account "massive" gravitons
From there we next will in future articles postulate conditions for experimental detectors for subsequent data sets to obtain falsifiable data sets.

Minimum scale factor, cosmological constant, space-time bubble, Penrose singularity

I. Begin first with a description of the emergent tunneling wave space-time equation used to compare the start of expansion, of that wave equation, with Planckian-era quantum conditions

We use the construction from [1] as to, if the initial 'potential' $V(\phi)$ is very large, how to isolate the form of the wavefunction, especially if $a^2V(\phi) > 1$, even if a is the initial value, i.e. very , very small, even if $a_{\min} \propto 5.13 \times 10^{-62}$, and then by page 269 of [1] go to the following formulation. Namely that we look at

$$\Psi_T \propto \frac{\exp\left(-1/3V(\phi)\right)}{\left(a^2V(\phi)-1\right)^{1/4}} \cdot \exp\left(-i \cdot \frac{\left(a^2V(\phi)-1\right)^{3/2}}{3V(\phi)}\right) \tag{1}$$

Our entries into all the above will be the subject of the next several sessions of our document and we will endeavor to explain how this all fits into the early universe modeling we will be working with as far as foundational research into Quantum gravity.

In terms of a contribution from the modified Einstein Equations, we will be considering the time components of the Einstein equation with the stress and strain component in quantum expectation language, i.e. as in Colins, Martin and Squires, written as, due to emergent space-time from pre Planckian to Planckian regimes. I.e. .look at the time related transitions, and this is what we get. I.e. the basic equations are related as follow

$$\left(\frac{R_{\mu\nu}-\frac{R}{2}\cdot g_{\mu\nu}+\Lambda g_{\mu\nu}}{8\pi G_N}+\langle\Psi|T_{\mu\nu}|\Psi\rangle=0\right)$$

$$\xrightarrow[\mu,\nu\to 0,0]{}\frac{R_{00}-\frac{R}{2}\cdot g_{00}+\Lambda g_{00}}{8\pi G_N}+\langle\Psi|T_{00}|\Psi\rangle=0 \tag{2}$$

Here we will have the following approximations, used,. Namely look at

$$\rho\approx\frac{\dot{\phi}^2}{2}+V(\phi)\equiv\frac{\gamma}{8\pi G}\cdot t^2+V_0\cdot\left\{\sqrt{\frac{8\pi G V_0}{\gamma(3\gamma-1)}}\cdot t\right\}^{\sqrt{\frac{\gamma}{4\pi G}}-\sqrt{\frac{8\pi G}{\gamma}}} \tag{3}$$

Which are a result of the following equations used. i.e.

Using this above is using Padmabhan's inputs into the inflaton potential and also into the inflaton itself, as given in [3]

which uses at the surface of a presumed non singular start to the expansion of the universe

$$a(t)=a_{\min}t^{\gamma} \tag{4}$$

Leading to [3] the inflaton.

$$\phi\approx\sqrt{\frac{\gamma}{4\pi G}}\cdot\ln\left\{\sqrt{\frac{8\pi G\cdot V_0}{\gamma\cdot(3\gamma-1)}}\cdot t\right\} \tag{5}$$

And what we will use later the "inflaton potential "we write as [5]

$$V=V_0\cdot\left\{\sqrt{\frac{8\pi G V_0}{\gamma(3\gamma-1)}}\cdot t\right\}^{\sqrt{\frac{\gamma}{4\pi G}}-\sqrt{\frac{8\pi G}{\gamma}}} \tag{6}$$

Furthermore, we have from [4] pages 212-3, and [5]

$$m\cdot\partial_t(a^2-a^3)=0 \tag{7}$$

And then, a minimum time step we define via a minimum time step of

$$t=\left(\frac{2}{3a_{\min}}\right)^{1/\gamma} \tag{8}$$

Note that if the time as defined by Eq. (8) is on the order of Planck time, i.e. 10^-44 seconds, we have then that $\gamma\approx 61-62$

We then close with a statement, if

$$\frac{3\left(\left(\frac{\ddot{a}}{a}\right)\cdot\left(g_{00}-1\right)+\left(\frac{\dot{a}}{a}\right)^{2}g_{00}+\frac{\kappa}{a^{2}}g_{00}\right)+\Lambda g_{00}}{8\pi G_{N}}$$

$$=-\left\langle\Psi\left|\frac{c\dot{\phi}^{2}}{2}+cV\left(\phi\right)\equiv\frac{c\gamma}{8\pi G}\cdot t^{2}+cV_{0}\cdot\left\{\sqrt{\frac{8\pi GV_{0}}{\gamma\left(3\gamma-1\right)}}\cdot t\right\}^{\sqrt{\frac{\gamma}{4\pi G}}-\sqrt{\frac{8\pi G}{\gamma}}}\right|\Psi\right\rangle \tag{9}$$

If t he wave function is taken via Eq. (1) put in, we have that the effective wave function for the "Quantum" part in the RHS of Eq. (9) is then

$$\Psi_{T}\propto\frac{\exp\left(-1/3V\left(\phi\right)\right)}{\left(a^{2}V\left(\phi\right)-1\right)^{1/4}} \tag{10}$$

Whereas we will be looking at the following approximation for the right hand side of Eq. (10), namely

$$\frac{R_{00}-\frac{R}{2}\cdot g_{00}+\Lambda g_{00}}{8\pi G_{N}}+\left\langle\Psi\left|T_{00}\right|\Psi\right\rangle$$

$$=\frac{3\left(\left(\frac{\ddot{a}}{a}\right)\left(g_{00}-1\right)+\left(\frac{\dot{a}}{a}\right)^{2}g_{00}+\frac{\kappa}{a^{2}}g_{00}\right)+\Lambda g_{00}}{8\pi G_{N}} \tag{11}$$

$$+\left\langle\Psi\left|\frac{c\gamma}{8\pi G}\cdot t^{2}+cV_{0}\cdot\left\{\sqrt{\frac{8\pi GV_{0}}{\gamma\left(3\gamma-1\right)}}\cdot t\right\}^{\sqrt{\frac{\gamma}{4\pi G}}-\sqrt{\frac{8\pi G}{\gamma}}}\right|\Psi\right\rangle$$

The above Eq(11) is set equal to zero.

Hence also note we are going to do the following for breaking down the parts of what would otherwise be an ENORMOUS integrand expression to evaluate. Namely the midpoint rule for numerical evaluation of the integral, using [6]. We will justify this procedure due to the "radius" of the initial "space-bubble" is so close to a singularity, and we are talking about emergent spacetime. Hence, this idea as of [6] is used whereas I would NEVER otherwise even think of it- The centering of the initial problem, about it's time component should also be seen in the context that the idea as will be used in further developments is that time, within the space-time bubble itself does NOT exist, and that there is instead a construction of time on the SURFACE of the initial near singularity.

The author is well aware of the Penrose singularity theorem [7]. At the end of this document, there will be a discussion of where I think it, (the singularity theorem) holds, and what is missed, which is part of the multiverse discussion which is explicitly put in in order to force the constant value of $\hbar$ cosmological cycle per cycle without referencing the insanity of the Anthropic principle which has been used by Tippler and others to insert a religious dimension into Cosmology, which I view as missing the point entirely as to the evolution of the Universe itself. [8]. This is different from an argument given by Tegmark, as of [9] which gives a surprisingly well motivated argument as to spatial dimensions. I think the argument is well worth investigating. But the Strong Antropic principle is quite another matter entirely, to be avoided.

$$\left\langle \Psi \left| \frac{c\gamma}{8\pi G} \cdot t^2 + cV_0 \cdot \left\{ \sqrt{\frac{8\pi G V_0}{\gamma(3\gamma-1)}} \cdot t \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \right| \Psi \right\rangle = A_1 + A_2$$

$$A_1 = \left\langle \Psi \left| \frac{c\gamma}{8\pi G} \cdot t^2 \right| \Psi \right\rangle \approx \ell^3_{Planck} \int dt \left\{ \frac{\exp\left(-2/3V(\phi)\right)}{\left(a^2 V(\phi) - 1\right)^{1/2}} \cdot \frac{c\gamma}{8\pi G} \cdot t^2 \right\}$$

$$A_2 = \ell^3_{Planck} \int dt \left\{ \frac{\exp\left(-2/3V(\phi)\right)}{\left(a^2 V(\phi) - 1\right)^{1/2}} \cdot cV_0 \cdot \left\{ \sqrt{\frac{8\pi G V_0}{\gamma(3\gamma-1)}} \cdot t \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \right\} \quad (12)$$

$$a = a_{\min} t^{\gamma}$$

$$V = V_0 \cdot \left\{ \sqrt{\frac{8\pi G V_0}{\gamma(3\gamma-1)}} \cdot t \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}}$$

From there we will use simple approximations to ascertain as to how to analyze Eq. (11) and Eq. (12) and proceed with the rest of this article mainly using the midpoint rule for numerical approximation

To do so, we look at the following approximation, namely if

$$A_1 = \left\langle \Psi \left| \frac{c\gamma}{8\pi G} \cdot t^2 \right| \Psi \right\rangle \approx \ell^3_{Planck} \int dt \left\{ \frac{\exp\left(-2/3V(\phi)\right)}{\left(a^2 V(\phi) - 1\right)^{1/2}} \cdot \frac{c\gamma}{8\pi G} \cdot t^2 \right\}$$

$$\approx \ell^3_{Planck} t_{Planck} \left\{ \frac{\exp\left(-2/3V\left(\frac{t_{Planck}}{2}\right)\right)}{\left(a^2 V\left(\frac{t_{Planck}}{2}\right) - 1\right)^{1/2}} \cdot \frac{c\gamma}{8\pi G} \cdot \frac{t^2_{Planck}}{4} \right\} \quad (13)$$

$$A_2 = \ell^3_{Planck} \int dt \left\{ \frac{\exp\left(-2/3V(\phi)\right)}{\left(a^2 V(\phi) - 1\right)^{1/2}} \cdot cV_0 \cdot \left\{ \sqrt{\frac{8\pi G V_0}{\gamma(3\gamma-1)}} \cdot t \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \right\}$$

$$\approx \ell^3_{Planck} t_{Planck} \left\{ \frac{\exp\left(-2/3V\left(\frac{t_{Planck}}{2}\right)\right)}{\left(a^2 V\left(\frac{t_{Planck}}{2}\right) - 1\right)^{1/2}} \cdot cV_0 \cdot \left\{ \sqrt{\frac{8\pi G V_0}{\gamma(3\gamma-1)}} \cdot \frac{t_{Planck}}{2} \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \right\} \quad (14)$$

Also

$$V\left(\frac{t_{Planck}}{2}\right)=V_0\cdot\left\{\sqrt{\frac{8\pi GV_0}{\gamma(3\gamma-1)}}\cdot\frac{t_{Planck}}{2}\right\}^{\sqrt{\frac{\gamma}{4\pi G}}-\sqrt{\frac{8\pi G}{\gamma}}} \qquad (15)$$

Also

$$a^2V\left(\frac{t_{Planck}}{2}\right)=a_{\min}^2 t^{2\gamma}\cdot V_0\cdot\left\{\sqrt{\frac{8\pi GV_0}{\gamma(3\gamma-1)}}\cdot\frac{t_{Planck}}{2}\right\}^{\sqrt{\frac{\gamma}{4\pi G}}-\sqrt{\frac{8\pi G}{\gamma}}} \qquad (16)$$

In addition the fact I will refer to a very strong initial domain wall, of a boundary between the early universe, and also state the following

Pick the following in order to insure the initial wavefunction as given by Eq.(1) remains valid within the context of a bubble of space-time at the start of inflation, namely have:

$$a^2V\left(\frac{t_{Planck}}{2}\right)=a_{\min}^2 t_{Planck}{}^{122}\cdot V_0\cdot\left\{\sqrt{\frac{8\pi GV_0}{61\cdot(183-1)}}\cdot\frac{t_{Planck}}{2}\right\}^{\sqrt{\frac{61}{4\pi G}}-\sqrt{\frac{8\pi G}{61}}}>1$$
$$\Rightarrow V_0^{1+\sqrt{\frac{8\pi G}{61}}-\sqrt{\frac{61}{4\pi G}}}>\frac{1}{\left\{a_{\min}^2 t_{Planck}^{122}\cdot\left\{\sqrt{\frac{8\pi G}{61\cdot(183-1)}}\cdot\frac{t_{Planck}}{2}\right\}^{\sqrt{\frac{61}{4\pi G}}-\sqrt{\frac{8\pi G}{61}}}\right\}} \qquad (17)$$

This restriction is in tandem with having t to the 61st power at the start of expansion of the Planckian space-time regime as reflecting the hyper rapid onset of inflation itself

II. Utilizing Eq. (2) and Eq. (3), as to nonsingular cosmology, and nonsingular Friedman Equations

To do this, we will attempt several goals, first of all, bring up what we can about the Cosmological Constant, within this framework and also give several reasons for

A given in our work is that within the nucleation of space-time, that time, even as given in Eq. (2) simply does not exist, but that we will be able to use the results of Freeze [10] as far as a nonsingular Friedman equation to come up with

$$H=\frac{\dot{a}}{a}=\sqrt{\frac{8\pi}{3M_P^2}}\cdot\sqrt{\left(\rho-\frac{\rho^2}{2|\sigma|}\right)} \qquad (18)$$

The easiest thing to do would be to make the following identification, namely writing a quadratic equation in terms of density, which we would write as the following form, namely for an unspecified for now g_{00} the following quadratic equation in terms of a general density function which we would write as

$$3\left(\left(\gamma\cdot(\gamma-1)t^{\gamma-1}\right)\cdot(g_{00}-1)+\frac{8\pi}{3M_P^2}\left(\rho-\frac{\rho^2}{2|\sigma|}\right)g_{00}+\frac{\kappa}{a^2}g_{00}\right)+\Lambda g_{00}$$

$$=-\left\langle\Psi\left|\frac{c\gamma}{8\pi G}\cdot t^2+cV_0\cdot\left\{\sqrt{\frac{8\pi GV_0}{\gamma(3\gamma-1)}}\cdot t\right\}^{\sqrt{\frac{\gamma}{4\pi G}}-\sqrt{\frac{8\pi G}{\gamma}}}\right|\Psi\right\rangle \tag{19}$$

Then we could write the following, namely

$$\left(\rho-\frac{\rho^2}{2|\sigma|}\right)+\frac{\Lambda\cdot M_P^2}{8\pi}+\frac{\kappa\cdot M_P^2}{a^2\cdot 8\pi}+\left(\gamma\cdot(\gamma-1)t^{\gamma-1}\right)\cdot\frac{(g_{00}-1)\cdot M_P^2}{g_{00}\cdot 8\pi}$$

$$+\left\langle\Psi\left|\frac{c\gamma}{8\pi G}\cdot t^2+cV_0\cdot\left\{\sqrt{\frac{8\pi GV_0}{\gamma(3\gamma-1)}}\cdot t\right\}^{\sqrt{\frac{\gamma}{4\pi G}}-\sqrt{\frac{8\pi G}{\gamma}}}\right|\Psi\right\rangle\frac{M_P^2}{g_{00}\cdot 8\pi}=0 \tag{20}$$

We will from here isolate the Cosmological constant, which we write as, to first order

Here , we have that ρ is a space-time density function, whereas σ is related to the tension of a space-time bubble presumably of the order of a Planck radius. And we are also using what is given in [7] as far as a Dark Energy model, which we write, for energy density, as given by, if $\Re$ is the Ricci scalar [8] and we use also the notation of [9]. We have also the discussion given in [10] and set κ as spacetime curvature, and so then we have

$$\rho_{DE}=\frac{3\tilde{\alpha}}{8\pi}\cdot\left(\dot{H}+2H^2+\frac{\kappa}{a^2}\right)\equiv-\frac{\tilde{\alpha}}{8\pi}\cdot\Re \tag{7}$$

In terms of the bubble of spacetime before inflation, we submit that time does not really exist and that then we will be considering a rewrite of the above as having, effectively, $\dot{H}$ =0. And the term $\rho_{DE}=\rho$. And if we apply Eq. (6), we have within the bubble of spacetime

$$\left(\frac{16\pi}{3M_P^2}\cdot\left(\rho-\frac{\rho^2}{2|\sigma|}\right)+\frac{\kappa}{a^2}-\left(\frac{3\underset{\sim}{\alpha}}{8\pi}\right)^{-1}\rho\right)=0 \tag{8}$$

Or, then a quadratic equation for $\rho_{DE}=\rho$ above will then become the following quadratic equation

$$\rho^2 - |\sigma|\left(2 - \frac{1}{\tilde{\alpha}}\right)\rho - \frac{3M_P^2|\sigma|}{8\pi}\frac{\kappa}{a^2} = 0$$

$$\rho^2 - |\sigma| \cdot \left(2 - \frac{1}{\tilde{\alpha}}\right) \cdot \rho - \frac{3M_P^2|\sigma|}{8\pi} \cdot \frac{\kappa}{a^2} = 0 \quad (9)$$

A simple quadratic equation solution for $\rho_{DE} = \rho$ in Eq. (9) yields the following

$$\rho \equiv |\sigma|\left(1 - \frac{1}{2\tilde{\alpha}}\right) \cdot \left(1 \pm \sqrt{1 + \frac{3M_P^2|\sigma|}{2\pi} \cdot \frac{\kappa}{a^2}}\right) \quad (10)$$

This value for $\rho_{DE} = \rho$ at just about the surface of a bubble of initial space-time nucleation , would be if κ were small but not zero would then be

$$\rho \simeq |\sigma|\left(1 - \frac{1}{2\tilde{\alpha}}\right) \cdot \left(1 \pm \left(1 + \frac{3M_P^2|\sigma|}{4\pi} \cdot \frac{\kappa}{a^2}\right)\right) \quad (11)$$

If or not we would have positive or negative energy density would weigh then upon the behavior of $\tilde{\alpha}$, which is what we will comment upon later in this manuscript.

Unlike the Wheeler De Witt formulation where there is a zero net "energy" value commensurate with no time evolution of the wave function of the universe, as seen in [11], with variants proposed in [12] and [13], we will be involved in using an input into the initial physical system for reasons brought up in [3], and this will allow us then to say something about the formulation of a time step at the surface of the space-time bubble, according to the minimum uncertainty principle, i.e. at the surface of the bubble, of space time, say of about one Planck length in radius, we would have then a **minimum timestep** defined by using[14]

$$\Delta E \Delta t = \hbar \quad (12)$$

If so then, if the initial volume is of the cube of a Planck length, we have a time step defined via

$$\Delta t \approx \frac{\hbar}{\left(l_{Planck-length}\right)^3 |\sigma|\left(1 - \frac{1}{2\tilde{\alpha}}\right) \cdot \left(1 \pm \left(1 + \frac{3M_P^2|\sigma|}{4\pi} \cdot \frac{\kappa}{a^2}\right)\right)} \quad (13)$$

For a sufficiently large $\tilde{\alpha}$ value, we would then have the following minimum timestep

$$\Delta t \approx \frac{\hbar}{\left(l_{Planck-length}\right)^3 |\sigma|\left(1 - \frac{1}{2\tilde{\alpha}}\right) \cdot \left(2 + \frac{3M_P^2|\sigma|}{4\pi} \cdot \frac{\kappa}{a^2}\right)} \quad (14)$$

If the $\tilde{\alpha}$ value, were instead small, we would be probably be looking t

$$\Delta t \approx \frac{\hbar}{\left(l_{Planck-length}\right)^3 |\sigma| \left|\left(1-\frac{1}{2\tilde{\alpha}}\right)\right| \cdot \left(\frac{3M_P^2|\sigma|}{4\pi} \cdot \frac{\kappa}{a^2}\right)} \qquad (15)$$

In either case, we will be doing our calculations to determine what this has to say as to the frequency of a signal from this event, as well as the strength of GW, and then also the possible polarization states. This would have to be contrasted with Eq. (5), as to what that says about bounding values for the input into Eq. (15)

Let us now refer to a simple but specific model given by Dr. Corda, as to the way one can implement an amplitude for GW [14] which the author views as commensurate with regards to amplitude for this initial system of GW generation at the start of the expansion of the Universe.

From [14] we have that there are minimal extensions of General Relativity for the existence of GW generation which may be thoroughly explored which will enable us to have GW early universe production. Before we get to that we will briefly review what is known about the GW generation of the classical kind which is a precursor to understanding the problems with GW amplitude calculations in the beginning of Space-time evolution

I. Brief review of the basics of GW amplitude calculations in the traditional GW theories

To do this, we look at [15] which is Maggiore's first volume masterpiece of GW data analysis and collation of data. In doing so, on page 211, we have that if a GW were signified to run in the Z direction, then in lowest order, assuming that we are using

$$\begin{aligned} h_+ &= r^{-1}Gc^{-3} \cdot \left[\ddot{M}_{11} - \ddot{M}_{22}\right] \\ &\equiv 2r^{-1}Gc^{-5} \cdot \int d^3x \left[\partial_0^2\left(\left(x_1^2 - x_2^2\right)\cdot T^{00}\right)\right] \\ h_\times &= 2r^{-1}Gc^{-3} \cdot \left[\ddot{M}_{12}\right] \\ &\equiv 2r^{-1}Gc^{-5} \cdot \int d^3x \left[\partial_0^2\left(\left(x_1 x_2\right)\cdot T^{00}\right)\right] \end{aligned} \qquad (15)$$

Here, we have that, usually, $h_\times \approx h_+$

In doing so, we can write up what we have here with respect to the Einstein Energy tensor as

$$T^{00} \equiv c \cdot \rho_{Energy-density} = \frac{c \cdot \left(R_{00} - g_{00}R\right)}{8\pi} \qquad (16)$$

Here by [16] we can write

$$R_{00} = 4h_+\ddot{h}_+ + 2\dot{h}_+^{\,2} + 4h_\times\ddot{h}_\times + 2\dot{h}_\times^{\,2} \qquad (17)$$

And the Ricci Scalar as

$$R = g^{uv}R_{uv} \qquad (18)$$

Whereas we can then proceed in making sense of what Eq. (16), Eq. (17) and Eq. (18) are saying in terms of Massive Gravity, and relic GW generation

The action will start in the interpretation of R_{00} Ricci tensor, time domain, and R as a Ricci scalar term

II. Interpreting R_{00} Ricci tensor, time domain, and R

[17] gives us a different view of the eq. (16) to Eq. (18) terms which we can summarize, below. Afterwards we will go to that procedure brought up by Corda in [14] in terms of relic conditions. Here, the Ricci scalar is given directly as

$$R = g^{uv} R_{uv} = 6\Lambda \tag{19}$$

Whereas the term, given in Eq. (17) gets a new paintjob which we can have as by Kolb and Turner [18] for a Roberson-Walker cosmology, if we stick initially with Eq. (17) would have the following equality

$$R_{00} = 4h_{+}\ddot{h}_{+} + 2\dot{h}_{+}^{2} + 4h_{\times}\ddot{h}_{\times} + 2\dot{h}_{\times}^{2} \approx -3 \cdot \frac{\ddot{a}}{a} \tag{20}$$

Whereas we would re write Eq. (19) as the following

$$R = g^{uv} R_{uv} = 6\Lambda \approx -6 \cdot \left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^{2} + \frac{2\kappa}{a^{2}} \right] \tag{21}$$

In our case we would consider the Eq. (17) to be dominant in our problem, and therefore choose Eq. (16) to read as

$$T^{00} \equiv c \cdot \rho_{Energy-density} = \frac{c \cdot \left(R_{00} - g_{00} R \right)}{8\pi} \approx \frac{c \cdot \left(-3 \cdot \frac{\ddot{a}}{a} - g_{00} \cdot 6\Lambda \right)}{8\pi} \tag{22}$$

i.e. of all things a negative energy density, which is weird, at the surface of the bubble of space-time, whereas if we use Eq. (21) directly we would have, instead

$$T^{00} \equiv c \cdot \rho_{Energy-density} \approx \frac{c \cdot \left(-3 \cdot \frac{\ddot{a}}{a} + g_{00} \cdot 6 \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^{2} + \frac{2\kappa}{a^{2}} \right) \right)}{8\pi} \tag{23}$$

Whereas then we would have the following to contend with, as far as

$$h_+ = r^{-1} G c^{-3} \cdot \left[\ddot{M}_{11} - \ddot{M}_{22} \right]$$

$$\equiv 2 r^{-1} G c^{-4} \cdot \int d^3 x \left[\partial_0^2 \left(\left(x_1^2 - x_2^2 \right) \cdot \left\{ \frac{\left(-3 \cdot \frac{\ddot{a}}{a} - g_{00} \cdot 6\Lambda \right)}{8\pi} \right\} \right) \right] \qquad (24)$$

$$\approx 2 r^{-1} G c^{-4} \cdot \int d^3 x \left[\partial_0^2 \left(\left(x_1^2 - x_2^2 \right) \cdot \left\{ \frac{\left(-3 \cdot \frac{\ddot{a}}{a} + \left[6 g_{00} \cdot \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 + \frac{2\kappa}{a^2} \right) \right] \right)}{8\pi} \right\} \right) \right]$$

Whereas we could write, the other magnitude as

$$h_\times = r^{-1} G c^{-3} \cdot \left[\ddot{M}_{12} \right]$$

$$\equiv 2 r^{-1} G c^{-4} \cdot \int d^3 x \left[\partial_0^2 \left(\left(x_1 x_2 \right) \cdot \left\{ \frac{-3 \left(\frac{\ddot{a}}{a} \right) - g_{00} \cdot 6\Lambda}{8\pi} \right\} \right) \right] \qquad (25)$$

$$\approx 2 r^{-1} G c^{-4} \cdot \int d^3 x \left[\partial_0^2 \left(\left(x_1 x_2 \right) \cdot \left\{ \frac{-3 \left(\frac{\ddot{a}}{a} \right) + g_{00} \cdot 6 \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 + \frac{2\kappa}{a^2} \right)}{8\pi} \right\} \right) \right]$$

These would have to be configured to be at the surface of the space-time bubble with the Time derivative, in this case, according to Maggiore to be retarded time, and also using Eq. (2) for the scale factor. Whereas we could put in different candidates as to the g_{00} term

III. Comparing these values of with the density values of Eq.(10) for density, as put in would lead to Eq.(24) and Eq. (25) re written in terms of the bubble of space-time with a base of Eq.(13) would remove the potential of Negative values.

Here is the problem if that is done. I.e. the answer would be dependent upon a reading, experimentally upon the tension of the bubble of Space-time presumed. Difficult but not impossible. But necessitating a huge amount of work. $|\sigma|$ would need experimental confirmation, whereas we would be using retarded time in the time differentiation.

$$
\begin{aligned}
h_{\times} &= r^{-1}Gc^{-3}\cdot\left[\ddot{M}_{12}\right] \\
&\equiv 2r^{-1}Gc^{-4}\cdot\int d^3x\left[\partial_0^2\left(\left(x_1 x_2\right)\cdot\left\{\ \right\}\right)\right] \\
&\approx 2r^{-1}Gc^{-4}\cdot\int d^3x\left[\partial_0^2\left(\left(x_1 x_2\right)\cdot\left\{\left|\sigma\right|\cdot\left(1-\frac{1}{2\tilde{\alpha}}\right)\cdot\left(1\pm\sqrt{1+\frac{3M_P^2\left|\sigma\right|\kappa}{2\pi a^2}}\right)\right\}\right)\right]
\end{aligned}
\tag{26}
$$

And

$$
\begin{aligned}
h_{+} &= r^{-1}Gc^{-3}\cdot\left[\ddot{M}_{11}-\ddot{M}_{22}\right] \\
&\equiv 2r^{-1}Gc^{-4}\cdot\int d^3x\left[\partial_0^2\left(\left(x_1^2-x_2^2\right)\cdot\left\{\ \right\}\right)\right] \\
&\approx 2r^{-1}Gc^{-4}\cdot\int d^3x\left[\partial_0^2\left(\left(x_1^2-x_2^2\right)\cdot\left\{\left|\sigma\right|\cdot\left(1-\frac{1}{2\tilde{\alpha}}\right)\cdot\left(1\pm\sqrt{1+\frac{3M_P^2\left|\sigma\right|\kappa}{2\pi a^2}}\right)\right\}\right)\right]
\end{aligned}
\tag{27}
$$

This though lacks the third term from an additional polarization term which was used by Corda in [14] which will be Referred to later. Before doing that, look at the additional development we could do, with Eq. (10) for a GW density term . We can though add in the cross polarization term before going to the Corda treatment in [14]It is his Eq. (48). Which is our Eq. (28)

$$
.\ h_{uv} \equiv A^{+}\left(t-z\right)\cdot e_{uv}^{(+)} + A^{\times}\left(t-z\right)\cdot e_{uv}^{(\times)} + h_m\left(t-v_G z\right)\eta_{uv}
\tag{28}
$$

IV. Linking to the Corda Approach in [14] for relic GW conditions, i.e. adding Equations together

The Corda approach, in reference [14] is in its Eq. (48) a representation we will refer to as setting, if we used the cosmological parameter referenced in Eq.

$$
\begin{aligned}
A^{\times}\left(t-z\right)\cdot e_{uv}^{(\times)} \leftrightarrow h_{\times} &\equiv 2r^{-1}Gc^{-4}\cdot\int d^3x\left[\partial_0^2\left(\left(x_1 x_2\right)\cdot\left\{\frac{-3\left(\dfrac{\ddot{a}}{a}\right)-g_{00}\cdot 6\Lambda}{8\pi}\right\}\right)\right] \\
A^{+}\left(t-z\right)\cdot e_{uv}^{(+)} \leftrightarrow h_{\times} &\equiv 2r^{-1}Gc^{-4}\cdot\int d^3x\left[\partial_0^2\left(\left(x_1^2-x_2^2\right)\cdot\left\{\frac{-3\left(\dfrac{\ddot{a}}{a}\right)-g_{00}\cdot 6\Lambda}{8\pi}\right\}\right)\right]
\end{aligned}
\tag{29}
$$

The missing term is in $h_m\left(t - v_G z\right)\eta_{uv}$. We will make some comments as to this additional term

V. Adding in $h_m\left(t - v_G z\right)\eta_{uv}$ and coming up with a primordial GW amplitude generation

To do this we will consider having a look at the Eq.(45) of reference [14] which we will write as

$$d\rho_{GW}(relic) \approx \hbar \frac{\left(H_{dS} H_0\right)^2}{4\pi^2 c^3} \frac{d\omega}{\omega} \tag{30}$$

We claim that this would be commensurate with the Hubble expansiton H_0 defined by setting it to today's space time crucial density

$$H_0^2(today) \equiv \frac{8\pi G \rho_c(critical - density)}{3c^2} \tag{31}$$

Whereas earlier we would have

$$H_{dS}^2(beginning) \equiv \frac{8\pi G \rho_{dS}}{3c^2} \tag{32}$$

Our approximation in our paper is to use the following substitution, namely make the following substitution

$$\rho_{dS} \Leftrightarrow \left\{ \frac{-3\left(\frac{\ddot{a}}{a}\right) - g_{00} \cdot 6\Lambda}{8\pi} \right\} \approx \left\{ \frac{-3\left(\frac{\ddot{a}}{a}\right) + g_{00} \cdot 6\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{2\kappa}{a^2}\right)}{8\pi} \right\} \tag{33}$$

Then to use an integrated version of this in Eq. (32) and then come up with a linkage to analyzing

$$\rho_{GW}(relic) \approx \hbar \frac{\left(H_0\right)^2}{4\pi^2 c^3} \int \frac{d\omega {H_{dS}}^2}{\omega} \tag{34}$$

Re write this as follows set

$$H_{dS}^2(beginning) \equiv \frac{8\pi G \rho_{dS}}{3c^2} \approx \frac{8\pi G}{3c^2} \cdot \left\{ \frac{-3\left(\frac{\ddot{a}}{a}\right) + g_{00} \cdot 6\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{2\kappa}{a^2}\right)}{8\pi} \right\} \tag{35}$$

Then

$$\rho_{GW}(relic) \approx \hbar \frac{(H_0)^2}{4\pi^2 c^3} \int \frac{d\omega}{\omega} \cdot \frac{8\pi G}{3c^2} \cdot \left\{ \frac{-3\left(\frac{\ddot{a}}{a}\right) + g_{00} \cdot 6\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{2\kappa}{a^2}\right)}{8\pi} \right\} \tag{36}$$

This would lead to a third relic condition GW contribution we would call

$$h_l(pure - relic) \approx 2r^{-1}Gc^{-4} \cdot \int d^3x \left[\partial_0^2 \left((x_1 x_2) \cdot \{ \rho_{GW}(relic) \} \right) \right] \tag{37}$$

VI. **What about** tensor-mode, vector-mode and scalar-mode gravitons **as spin zero and spin 1 "gravitons"?**

We will review our work further but it will be in strict fidelity with what Wen Hao and Fangyu Li did, which is in ARXIV and was just recently published in European Journal C, i.e. see this [19].

Reference [19] is unusually complete. What the author is arguing for is that since the details of the scalar and vector mode interactions are highly detector specific , that adding in these 2 additional polarization states, akin to the five polarization states of massive gravitons will await fine tuning the details of the material in this document, as to the first three polarizations, as well as getting more details as to the instrumentation of the Li Baker detector. This will be followed up upon, pending utilizing the details as to the following document [20] , [21]

Not that this is alluded to, as far as future work, we will go to the issue of relic GW frequencies

VII. Order of magnitude estimates for Frequency, say of relic GW

We use the most direct route toward doing it, and say

$$\omega \approx |\sigma| \left(l_{Planck-length} \right)^3 \left(1 - \frac{1}{2\tilde{\alpha}} \right) \cdot \left(1 \pm \left(1 + \frac{3M_P^2 |\sigma|}{4\pi} \cdot \frac{\kappa}{a^2} \right) \right) \tag{38}$$

For sufficiently large $\tilde{\alpha}$ value

$$\omega \approx \left(l_{Planck-length} \right)^3 |\sigma| \left(1 - \frac{1}{2\tilde{\alpha}} \right) \cdot \left(2 + \frac{3M_P^2 |\sigma|}{4\pi} \cdot \frac{\kappa}{a^2} \right) \tag{39}$$

If instead we have a small $\tilde{\alpha}$ value

$$\omega \approx \left(l_{Planck-length} \right)^3 |\sigma| \left| \left(1 - \frac{1}{2\tilde{\alpha}} \right) \right| \cdot \left(\frac{3M_P^2 |\sigma|}{4\pi} \cdot \frac{\kappa}{a^2} \right) \tag{40}$$

These expressions should be compared to Eq (5) for which we can write, say[14]

$$t = \left(\frac{2}{3 \cdot a_{\min}} \right)^{1/\gamma} \Leftrightarrow E \equiv \hbar\omega \Rightarrow \omega \approx \left(\frac{2}{3 \cdot a_{\min}} \right)^{-1/\gamma} \qquad (41)$$

We think that this means that at the boundary of a space time bubble, that this would force us to have an enormous value for γ. We would also, if the initial energy were just at the boundary of the bubble of space time, have the odd situation for which we would have the following, namely at the surface of the bubble,

$$V_0 \cdot \left[\frac{V_0 \cdot 8\pi G}{\gamma \cdot (3\gamma - 1)} \cdot \left(\frac{2}{3 \cdot a_{\min}} \right)^{-/\gamma} \right]^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \approx E \equiv \hbar\omega \approx \hbar \cdot \left(\frac{2}{3 \cdot a_{\min}} \right)^{-1/\gamma} \qquad (42)$$

To put it mildly, we would need a lot more experimental data sets!

All this leads to considerable interplay between the equations given in Eq. (38) to Eq. (42) We will discuss this in our conclusions. Keep in mind that the interplay with all of this, we will try to right after the Bubble of initially nucleated Space-time to have a linkage to simpler Quantum mechanics involved as seen in [22] . In addition we wish to in doing so to have fidelity with the insights of Christian Corda, as seen in [23] as to mixed Scalar-Tensor modes and $f(R)$ models of Gravity.

In our write up of Eq. (38) to Eq. (42) we are assuming for all general purposes that $a_{\min} = a$, Keep in mind that the surface of the bubble would be, in fidelity with reference [3] be obeying $a_{\min} = a$ and that in line with inflationary e folds of 60 [24] [15], or 1.14 times 10^26. If this is kept in mind, and say that we have a frequency range of about 10^37 Hertz, as a result of the above , we would then have say 10^11 Hertz for Earth signals taken for GW detection

VIII. What is the strength of a signal for our model?

GW signals have a simple strength of GW **moniker [24], [25]** The simplest idea is to look at the behavior of GW from a massive black hole, and in dong so, the strain is simply M the mass of a black hole, , r = distance of source to Earth from Black hole,

$$h \approx \frac{GM}{c^2} \times \frac{1}{r} \times \left(\frac{v}{c} \right)^2$$

$$with \quad v \quad speed \quad of \quad masses(system) \quad (21)\ (43)$$

$$w.r.t \quad light \quad speed$$

If the original universe were acting say as a black hole just before the onset of the black hole exploding via the violence of inflation , if we wish to have an observed GW signal, and have say v/c about 1, in terms of the early universe, with r being the distance of the expansion in 13.6 billion years , we could have a configuration for which if we were near the initial start point of the Universe, that Eq. (43) would be of the order of 1, whereas, the e fold expansion of 10^26 from an initial start point would then have say a signal strength of the order of 10^-25 to 10^-26, and if so a strain value of 10^-26 would be commensurate with GW signal detectors being designed as of Earth orbit where we are right now. Verifying though the initial configuration of Eq. (43) as approaching unity, though, would require serious investigative work and would entail perhaps spending attention to the fractal geometry given in [26] [17]

The entire reference in [26] [17] is essentially a referencing of the need for fractal dimensionality, and where it fits in, as far as the geometry of space-time. On page 94 [26] of [17] the **Hausdorff-Besicovitch** dimension, which is fractal is done, in gory detail. Why this is important? Note that in [3] we made reference to [27][18], which if d(dimension) is fractal, means that the temperature for the universe, so assumed, has many surprises if we wish to connect Eq. (22) (44) with a scaling of $\hbar \cdot \omega$ i.e. there is then likely not a simply linear relationship between temperature, and frequency, hence we have to be more careful.

$$\Delta E = \frac{d(\dim)}{2} \cdot k_B \cdot T_{universe} \qquad (44)$$

Now, make use of [3] again and this relationship below though will stand the test of time, but it is NOT dependent upon frequency! Here N_g is a count of gravitons which may commence from our earlier generation of early universe GW. From [3]

$$\begin{aligned} &\left[N_g / Vol(in-Planck-units) \right] \sim s(entropy-density) = \frac{2\pi^2}{45} g_* \cdot \left(T_{universe} / T_{Planck} \right)^2 \\ &\Rightarrow \left(T_{universe} / T_{Planck} \right)^2 \approx \left(\frac{2\pi^2}{45} g_* \right)^{-1} \cdot \left[N_g / Vol(in-Planck-units) \right] \end{aligned} \qquad (45)$$

In other words, due to fractal geometry, while Eq. (22) and Eq. (23) may hold, as stated in [3] , we may have some observational surprises as far as frequency spectrum data sets as far as GW from relic initial GW generating events in the early universe.

In other wise, while the scaling of 10^-26 downward in terms of general frequency from a nonsingular universe starting point is likely correct, the details of the frequency spectrum in the modern era from Primordial beginnings may be a bit different from what we think.

IX. Is there another way to form a quantum wavefunction of the Universe rather than Wheeler De Witt ?

In [27] there is reference to a solution for which we still have

$$H(hamiltionian) \cdot \Psi(universe - wavefcn) = 0 \qquad (46)$$

Whereas we have, $\kappa \equiv \pm 1, 0$ and

$$H(hamiltionian) = \frac{3\pi c^2}{4G} \cdot a^2 \cdot \left(\frac{4G^2 P_a^2}{9\pi^2 c^4 a^4} + \frac{\kappa c^2}{a^2} - \frac{8\pi G \rho}{3c^2} \right) \qquad (47)$$

Whereas [27] sets ρ as an energy density, for a whole slew of grab bag assorted “topics”, and

$$P_a^2 \to \frac{\hbar^2}{a^{p^*}} \partial_a \cdot \left(\partial_a \right) \qquad (48)$$

Here, p^* is a so called Order parameter, and after a small scale factor solution we would get a linear combination of Bessel and Hankel functions for the wavefunction of the universe, which for small scale factor allowed to go to zero we have

$$\Psi(universe-wavefcn)\xrightarrow[a\to 0]{}\left(c_1+c_2a^{1-p^*}\right) \qquad (49)$$

Here, c_1 and c_2 are constants set within [27], but the entire Eq. (49) demands that we can shrink $a\to 0$

I.e. whatever time dependence this document has, in Eq. (49) would be through its scale factor, covertly speaking whereas there is no provision to take into a minimum value of the scale factor not equal to zero, as seen in Eq,. (5) and also no minimum time step ascertained when we wrote , assuming $a=a_{\min}$ and NOT going to zero, whereas also $\kappa>0$, and also $\left(E_{\min}=\hbar\cdot\omega_{gw}\right)\Delta t\approx\hbar$

$$\Delta t\approx\frac{\hbar}{\left(l_{Planck-length}\right)^3|\sigma|\left|\left(1-\frac{1}{2\tilde{\alpha}}\right)\right|\cdot\left(\frac{3M_P^2|\sigma|}{4\pi}\cdot\frac{\kappa}{a_{\min}{}^2}\right)} \qquad (50)$$

Hence

$$\omega_{gw}\approx\left(l_{Planck-length}\right)^3|\sigma|\left|\left(1-\frac{1}{2\tilde{\alpha}}\right)\right|\cdot\left(\frac{3M_P^2|\sigma|}{4\pi}\cdot\frac{\kappa}{a_{\min}{}^2}\right) \quad (51)$$

So, how could we write in a minimum time step whereas we also have a quantum function ?

I will make a suggestion as such, whereas, it may be not accepted. But this is my candidate. From Gasiorowicz [14], for a wave function just outside the minimum scale factor, I would try using Eq(50) and Eq.(51) to come up with a wavefunction at the surface of the space-time bubble which would have much the same information as given in Eq. (27) above. To do this though we will refer to a result by Shestakova [28], and [29] , and then re interpret it via some physics from Stephen Gasiorowicz [14] and compare that directly with Eq. (46), and Eq. (47)

X. Simple QM wavefunction at the surface of the Space-time non singular start point matched against Eq. (27)

We begin with [28] and [29] by T. P. Shestakova where we have on its page 9, if we use order parameter p=1, the following S.E. and also a solution, which is dependent upon a scale factor with $\tilde{\lambda}^*$ a dimensional factor put in as a fudge factor

$$\frac{d^2\Psi_{specialized}}{da^2}+\frac{1}{a}\frac{d\Psi_{specialized}}{da}-\tilde{\lambda}^*a^2\Psi_{specialized}=0 \qquad (52)$$

If so then, we will be looking at, using Arfken, 4th edition, page 667 [30] so that the solution to Eq. (52), given by, [28] and [29] has the form

$$\Psi_{specialized}(a)=\tilde{C}K_0\left(\frac{\tilde{\lambda}^*a^2}{2}\right)\approx\tilde{C}\cdot\left\{-\ln\left(\frac{\tilde{\lambda}^*a^2}{2}\right)-\tilde{\gamma}+\ln 2\right\} \qquad (53)$$

Note that the CRC handbook, [31] gives us an expansion so we can do the following

$$\Psi_{specialized}(a) \approx \tilde{C} \cdot \left\{ \left(1 - \frac{\tilde{\lambda}^* a^2}{2}\right) - \frac{1}{2}\left(1 - \frac{\tilde{\lambda}^* a^2}{2}\right)^2 + \dots - \tilde{\gamma} + \ln 2 \right\} \qquad (54)$$

Here we will set $a = a_{\min}$ and so then, if $a = a_{\min}$ is very small whereas we take $\tilde{\lambda}^* \to 1$ we would have

$$\Psi_{specialized}(a_{\min}) \approx \tilde{C} \cdot \left\{ \left(1 - \frac{a_{\min}{}^4}{8}\right) + \dots - \tilde{\gamma} + \ln 2 \right\} \qquad (55)$$

Leading to a comparison between two forms of the Wheeler De Witt equation give the same information , i.e. questioning if the following are equivalent

$$\begin{aligned} &\Psi_{specialized}(a_{\min}) \approx \tilde{C} \cdot \left\{ \left(1 - \frac{a_{\min}{}^4}{8}\right) + \dots - \tilde{\gamma} + \ln 2 \right\} \\ &\rightleftarrows \left(c_1 + c_2 a_{\min}{}^{1-p^*}\right)? \end{aligned} \qquad (56)$$

If so, and the answer is yes then we will then have to address what a minimum bubble of space-time ascertains as to future developments of space-time, and for this we will be looking at [14] for inspiration

XI. How to link Eq. (56) to a Schrodinger type equation for Planck time physics.

The relationship can be ascertained as to stating that we can use right at the surface of the space-time bubble

$$\begin{aligned} &\Delta t = \left(\frac{2}{3 \cdot a_{\min}}\right)^{1/\gamma} \Leftrightarrow \Delta E \Delta t \approx \hbar \Rightarrow \Delta E \approx E_{\min} \equiv \hbar \left(\frac{2}{3 \cdot a_{\min}}\right)^{-1/\gamma} \\ &\therefore \Psi_{Schrodinger}(wave - fctn - universe) \approx e^{iE_{\min} t} \propto \exp\left(i\hbar \left(\frac{2}{3 \cdot a_{\min}}\right)^{-1/\gamma} t \right) \end{aligned} \qquad (57)$$

Keep in mind that this is dependent upon a linkage to Eq(34), which has no explicit time dependence whereas Eq. (57) has a lot of time dependence

XII. And what about polarization states, especially if we have massive gravitons ?

See [32][23]. Our supposition is that as given in [32][23] that the massive gravity signature we would seek would be commensurate with " pure longitudinal and transverse breathing polarizations in the massive Horndeski theory and f(R) gravity". The details of such are given in their [32][23] document as given just above their Eq. (34) whereas we would have us calculate the electric component R (tjtk) of the Riemann tensor, as given in their document on page 10 of their arXIV document.

The details of their Eq. (34)_as in reference [32] would in the end have to be made fiducially relevant to the experimental platform as selected by an experimental gravity research team, and in itself would require a massive summary of known and sought after

We have already outlined a procedure tying in what we did with [14] as given by Dr. Corda. We intend to find Graviscalar and Gravivecor contributions, in line with work presented which is in fidelity with quantum gravity states as summarized already in this document.

XIII. Summing up. And a path forward

A, I am expecting a strain of Gravitational wave strength of h ~ 10^ -25 to 10^ -26

B. A frequency range for GW detected in Earth orbit of about 10^10 Hertz, which could be commensurate with initial GW of 10^36 to 10^ 37 Hertz for reasons which are in this manuscript. There would be due to e fold values of 60 a dramatic drop in frequencies.

C. The polarization states to watch, indicating if possible, would be for massive gravitons

D. Should there be a linkage between an initial worm hole style start to the formation of the observable universe, as a compliment to the big bank / inflation metrics, the author strongly recommends a review of the article given by Bruno J. Barros1, ∗ and Francisco S. N. Lobo as to relevant Wormhole geometry, [33] and the question of negative energy states in a wormhole threat. If as may be the case in nonsingular starts to the expansion of the universe, negative energy states make an appearance, this may be a clue as to what is called $f(R)$ geometries which in turn may have relevance to some of the polarization states in [32] The wormhole analogy may be crucial as to looking at the linkage between super massive black holes and recycling of the universe, which may be investigated later.

E. Also the author recommends a doubling down on the physics of this diagram in the last [3] article

See figure 1 below which is in [3]. We argue that the initial physics, may help us understand more of the self replication phenomena so outlined.

We also should understand if the physics of a linkage between Eq. (33) and Eq. (34) play a role as well

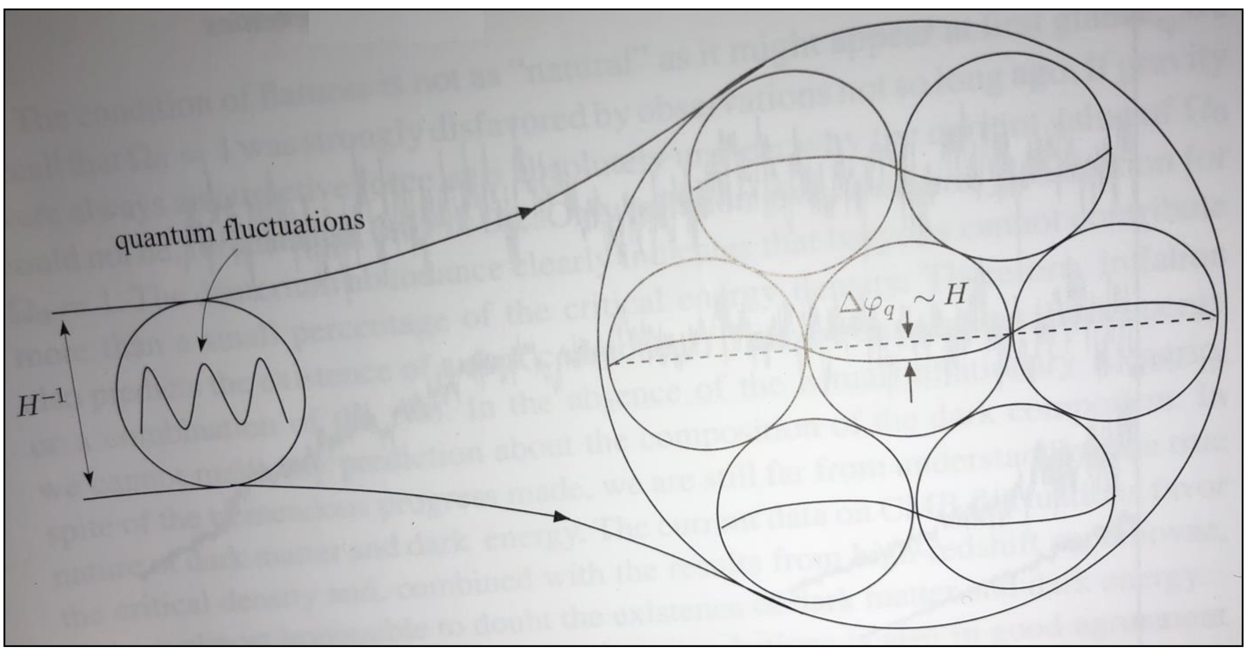

Figure 1 which is from reference [3]

F. Finally, and not least is to see if our feed into Eq. (33) to Eq. (34) and linkages to Figure 1, above, have in their genesis, the methodologies discussed in [34], i.e. as brought up in Phase transitions and the renormalization group as brought up by Jean-Zinn-Justin, which has a very interesting discussion of the Nancy Kerrigan problem, i.e. as of page 133 of [34] which states that DE did NOT interfere with , or put constraints upon structure formation. We submit that Figure 1 is a case of the DE density, initially being replaced as we did earlier, in the manuscript with cosmological energy density, using the equations we copied from Gong, et.al. [32] . This requires further investigation, and may if resolved have a tie in to the Figure 1 physics, of [3]

G. Doing all the above, would be hard, but would be the cornerstone of introducing the concept of using the construction of a **non-Archimedean field** to physics [35], which would be a way also to take into account what may be fractal geometry incorporated into space-time physics. In mathematics, a non-Archimedean ordered field is an ordered field that does not satisfy the Archimedean property. Examples are the **Levi-Civita field**, the hyperreal numbers, the surreal numbers, and the field of rational functions with real coefficients with a suitable order.

This would be a way to understand more of Eq. (22), if the dimensional coefficient, d, is representing fractal geometry.

VII. And now for some very specific goals for the future

We have referenced several different polarization methods for massive gravitons. Our idea is to form a bridge between our extension between [14] as a minimal extension of relativity and the methods brought up by Dr. Hao Wen, Dr. Fangyu Li, and others recently as published in European physics Journal C, given by [19] with an eye toward coming up with specific counter parts to the work of Gong , for massive gravitons [32]. The work in [19] is outstanding but in a sense very general, as we will try to look at what polarization states survive the onset of quantum gravity conditions at our nonsingular start to the universe

In addition, the author at the 4th Zeldovich conference, November 11, at 14:00 CET time made reference to the enduring mystery of error bars in the CMBR measurement as discussed by Abhay Akshenkar [36] of how his loop quantum gravity may form a solution to this problem. My entire nonsingular start to the evolution of the universe was initiated as a way to attack this problem addressed by Abhay and I will endeavor to do more on that in a future publication.

Keep in mind that there are 61 or so e folds from the start to the finish of inflation, allowing the universe to expand to the size of a large grapefruit, and that the total amount of expansion in 13.4 billion years of evolution is about 141 e folds.

Finally Karen Freeze in Zeldovich 4 [37] spoke on the physics of massive Dark Stars which she said may contribute to supermassive black hole formation in the center of elliptical galaxies. This insight will be part of an extension of our work into gravitational astronomy and will be referenced as well in a future publication.

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